Möbius Filters and Resonators

Jeffrey M. Pond

Naval Research Laboratory, Washington, DC 20375-5347, USA

Abstract **— Resonator and filter topologies are introduced which rely on the geometric deformation of a guiding structure resulting in the establishment of resonant conditions in more compact structures. Conceptually, the structures are analogous to Möbius strips. The path length of the edge in a Möbius strip is twice an length of the edge of the rectangle deformed in creating the Möbius strip geometry. Analogously, this means the structure can be at resonance even though the length of the line, before deformation, was a half wavelength long. The concept has been demonstrated in planar geometries and in wire-loaded cavity structures. The dual-mode nature of the fundamental resonance is readily apparent.**

I. Introduction

The concept introduced is to distort the geometry in such a fashion that the geometry contributes to the phase change. In particular, we will discuss resonator geometries where the phase change across the structure to establish a resonant condition has contributions from both the wavelength as well as a deformation of the resonator geometry. In principle, a substantial decrease in resonator volume can be realized. The validity of the concept has been demonstrated by realizing both a planar dual-mode filter and a wire-loaded cavity.

In particular, resonators are studied where the deformation of the geometry contributes half the phase change required to achieve a resonant condition. These structures are referred to as Möbius resonators and filters because of the obvious similarity to the Möbius strip. A practical definition [1] of a Möbius strip is "a one-sided surface that is constructed from a rectangle by holding one end fixed, rotating the opposite end through 180 degrees, and applying it to the first end". The Möbius strip is probably the most commonly cited geometric shape used to illustrate topology [1]; "a branch of mathematics concerned with those properties of geometric configurations which are unaltered by elastic deformations that are homeomorphisms". The deformation of the rectangle that takes place in forming a Möbius strip is a rotation of the geometry through 180 degrees.

With the exception of certain hybrid ring or "rat-race" type structures [2]-[4] the concept of twisting or deforming the geometry to increase the total phase shift has not been widely applied to distributed electromagnetic circuit. Although not contained in the definition above, in addition to being a one-sided surface, the Möbius strip possesses only one edge. The path length of this edge is equal to 4π times the mean radius of the Möbius strip. Exploiting this periodicity allows a resonance condition to exist in a smaller volume than can be realized without the geometric deformation.

Since the Möbius strip, by definition, contains a 180-degree deformation of the geometry, the distributed electromagnetic analog is expected to require a "circumference" corresponding to a half wavelength so that when combined with the additional 180 degrees of geometric deformation, a resonant condition exists.

At a particular frequency, f, if a section of transmission line a half wavelength long were bent back on itself, resonance could not occur as the field would not match. Indeed the wave at both "ends" would be 180 degrees out of phase. The line would need to be a full wavelength long to yield a resonant condition (Alternatively, it would have a resonant condition at a frequency of $2f$). If, however, the half-wavelength section of transmission line were to undergo a deformation that introduced a 180-degree phase reversal, the resonant condition would be satisfied. The volume of such a resonator would be greatly reduced. Its circumference would be one-half and its volume would be one-fourth, assuming the height was the same for both cases.

Figure 1 graphically demonstrates the concept. Figure 1a shows the sinusoidal pattern of a wavelength of a signal on a transmission line. The horizontal axis is labeled in Degrees, but is also directly proportional to the physical length. If the length of the transmission line is smoothly bent around on itself such that **A** connects to **A'** and **B** connects to **B'**, then the sinusoidal wave pattern matches which is consistent with resonance. In contrast, in Figure 1b, the length of the line is only a half wavelength long. It is easily seen that by employing the same procedure as in Figure 1a and bending the line back on itself such that **A** connects to **A'** and **B** connects to **B'**, then the sinusoidal pattern does not match and a resonance condition does not occur at this frequency.

However, consider a section of transmission line that is of equal length to that shown in Figure 1b and perform a twisting of the one end by 180 degrees before applying it to the first end. It can be seen, as shown in Figure 1c, that the resonance condition is met since **A** connecting to **A'** and **B** connecting to **B'** results in the sinusoidal pattern being smoothly joined. Figure 1d shows a photograph of the strips shown in Figures 1a and 1c where the two ends are bent around in the manner discussed. Note that the sinusoidal pattern is continuous for both.

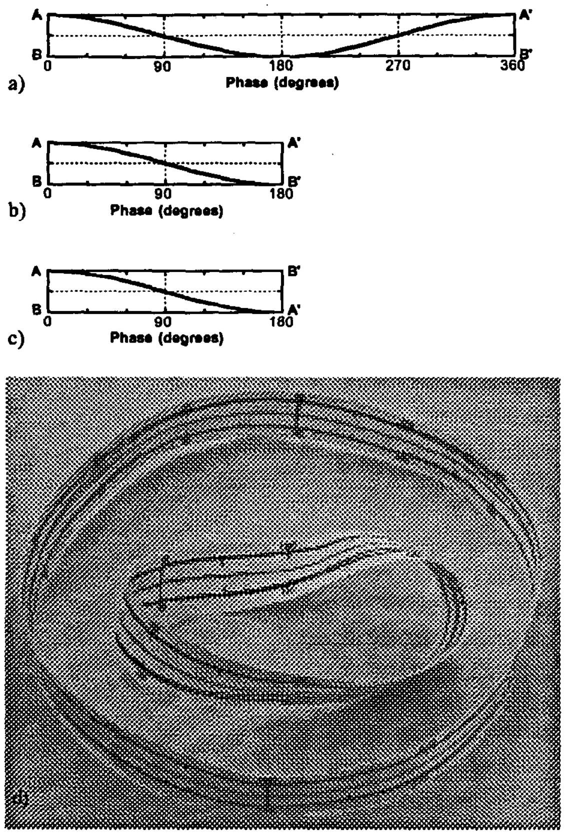

Fig. 1. When A–A' and B–B' the field pattern (red) of (a) resonance for a "ring-type" resonator, (b) anti-resonance for a "half-length" ring, (c) resonance for a "half-length" ring with a geometric 180° "twist". (d) 3-D representation of (a) and (c).

II. Planar Dual-Mode Resonators and Filters

It was decided to construct a Möbius twist resonator in a fashion that was consistent with realizing a planar microwave bandpass filter. The approach chosen began with a Teflon substrate that is copper clad on both sides. Unlike a microstrip circuit, both sides must be patterned to realize the required geometry. The main portion of the resonator is composed of the two nearly closed "C"s; one on each side of the substrate. The same photolithographic mask is used to transfer the pattern onto both surfaces. If the backside metallization pattern could be viewed through the substrate, it would appear to be a mirror of the metallization pattern that can be seen in Figure 2. Standard photoresist processing is used followed by etching the copper in ferric chloride solution. The patterning is done such that the "open" portion of the "C"s are aligned. The mean diameter was 3 cm. If, instead, the configuration was a standard ring resonator geometry (without the opening) the fundamental mode would occur when the mean circumference was one wavelength. The phase velocity reduction due to the Teflon ($\varepsilon_r = 2.25$) is such that the phase velocity is approximately 2.0×10^{10} cm/sec. The expected resonant frequency for the fundamental mode of a conventional ring resonator would be approximately 2.1 GHz.

To implement the geometric deformation or "twist", two via holes that connect the conductors on both sides of the substrate were drilled in the substrate. The "twist" results from inserting short wires through the Teflon substrate and soldering the wires to the top and bottom conductor. This can be seen at the top of Figure 2 where the vias are located at the ends of what appear to be a single interdigitated finger pair.

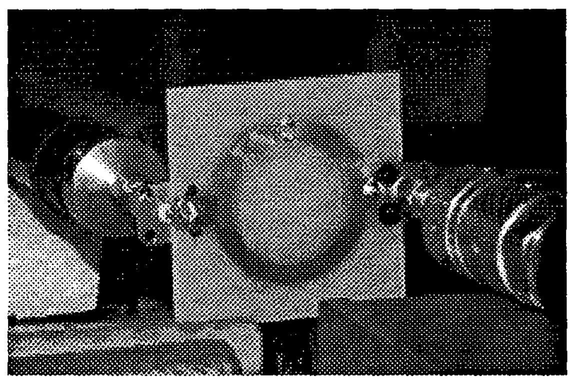

Fig. 2. Photograph of the first planar structure fabricated. All input and output coupling is capacitive. The "twist" is accomplished by two vias at the top of the loop. Both vias connect to a pattern on the reverse side printed with the same mask.

The completed resonator circuit is shown in Figure 2 connected to standard microwave test cables. The very short input and output transmission lines were placed 180 degrees apart on a diagonal of the resonator. The "twist" was placed at 90 degrees to the input and output transmission lines. Since simple gap coupling could not be controlled to sufficient accuracy, a thin dielectric was overlaid on the resonator so that the input and output lines could overlap with the resonator without making electrical contact. The coupling was adjusted by controlling the distance to which the input and output line metal traces overlay the resonator. This type of capacitive coupling was employed at the input and output ports for both the center conductor and outer conductor of the coaxial cable. SMA connectors were attached to both ports. The measured results are shown in Figure 3. As expected, the resonance/pass band is centered near 1 GHz. Both modes are distinctly evident in the reflection curve and can even be seen in the shape of the passband.

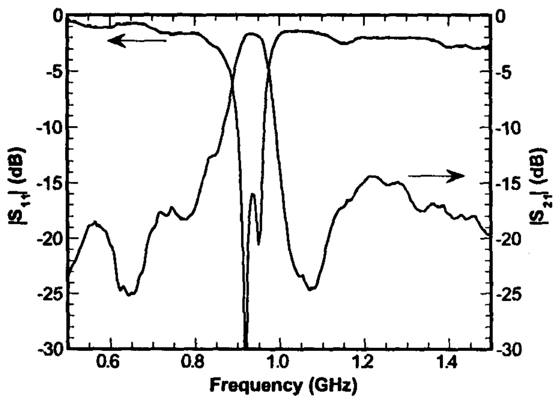

Fig. 3. Measured performance of the planar structure shown in Fig. 2. Although only a "proof-of-principle" construction, the shape of the return loss curve clearly demonstrates the dual-mode nature of the resonance.

III. Wire-Loaded Cavity Resonators

Another experimental implementation of this concept was to apply it to wire-loaded cavities. Helical wire-loaded cavities are well known for their size advantage in comparison to empty cavities [5]-[6]. The fundamental frequency of a helical wire-loaded cavity does not possess two degenerate modes. However, the fundamental frequency of a Möbius-wire-loaded cavity is expected to consist of two degenerate modes and thus may have some advantages in the realization of wire-loaded cavity filters.

The cavity used was a hollow cylindrical copper cavity with a diameter of 2.5 cm and a height of 1.5 cm. The cavity was split at approximately half its height and was clamped together with four screws. Small loop coupling antennas were fabricated from 0.085 inch diameter coaxial cable for both the input and output ports. These were mounted with a clamping arrangement that allowed the strength of the coupling to be varied at each port by sliding the loop further into the cavity. The fundamental frequency of the unloaded cavity was measured to be 8.94 GHz.

Several wire structures were fabricated using 0.141 inch diameter coaxial cable from which the outer conductor had been removed. The dielectric was retained as it allowed the separation between the conductors to be well controlled. As this would add some dielectric loading to the cavity as well as the loading due to the geometry of the wire, several control wire geometries were fabricated to compare to the Möbius-wire loaded cavity. The Möbius wire-loading geometry was hand formed with special attention to keeping the mean circumference (the line formed by the point of contact of the dielectric sheath) round and the geometric distortion distributed evenly around the circumference. Thus, unlike the planar implementation discussed previously, the geometric deformation is distributed uniformly along the circumference. In this respect, this wire structure is a more accurate realization of the common image of a Möbius strip with the wire representing the edge.

The Möbius-wire geometry was fabricated from a 10.16-cm length of wire that can be thought of as the edge of the Möbius strip. As this particular geometry is analogous to Figure 1c), it was determined that an appropriate control wire geometry should be analogous to the structure in Figure 1a). Of course, to fit within the cavity, it should have the same circumference and, hence, was composed of two round loops, each of which was 5.08 cm in length. Since the mean circumference was the same as the Möbius wire structure, it is expected that the fundamental resonance would be at about twice the frequency measured for the Möbius wire geometry. This fundamental resonance should, of course, also exhibit dual-mode properties due to the symmetry involved. The wire structures were suspended in the middle of the cavity using low-density low-loss semi-rigid foam supports. These were, also, hand formed so that the placement within the cavity was not held to very high tolerances.

The measured results for these two wire loaded cavity structures are shown in Figure 4. As can be readily seen the Möbius-wire-loaded cavity exhibits dual-mode behavior with a fundamental frequency slightly above 2 GHz. The degeneracy has, of course, been split and the two modes are at 2.15 and 2.2 GHz for a separation of 50 MHz. In comparison the parallel-loop-loaded cavity control resonator has a fundamental dual-mode resonance at 4.39 GHz and 4.48 GHz, slightly over 100 MHz apart. These results were obtained with the adjustment of the input and output coupling loops being the only variables available. Future efforts will include the use of tuning screws to adjust the coupling between modes and to vary the rotational angle of the "twist" cross-over with respect to the input and output ports. Also shown in Figure 4 is the 8.94 GHz fundamental mode of the unloaded cavity.

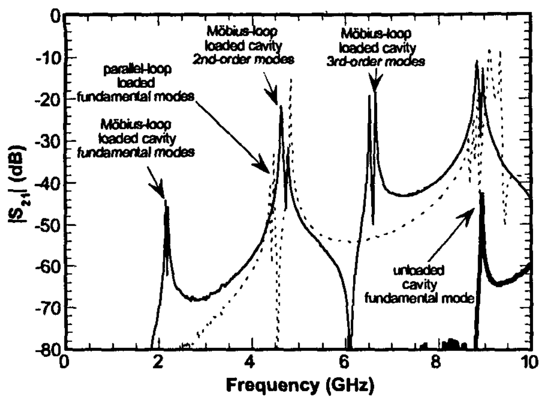

Fig. 4. Measured performance of a Möbius loop loaded cylindrical cavity (solid) in comparison to a parallel loop loaded cavity (dashed) with the same total wire length. The dual mode nature of the fundamental Möbius-loop mode is readily apparent. The dominant mode of the unloaded cavity is also shown (heavy solid).

Although not shown if Figure 4, one additional wire-loaded cavity was examined as a control. A two-turn helix was fabricated from the same dielectrically sheathed conductor. The length of the wire was 10.16 cm, identical to the total length of the Möbius-loop geometry. The fundamental single-mode resonance of this structure was at 900 MHz. This is consistent with what one should expect since, to first order, the resonance of the helical-loaded structure should correspond to length of the helix being at a half wavelength.

Given the same wire length, the helical wire-loaded cavity has a fundamental resonance at about half the frequency of the Möbius-loop wire-loaded cavity. However, the Möbius-loop wire-loaded cavity has an inherent dual-mode fundamental resonance, unlike the helical wire-loaded cavity fundamental mode.

IV. Conclusion

The concept of employing geometric deformations in order to contribute to the phase change necessary to achieve a resonance condition has been successfully demonstrated in both planar and wire-loaded cavity formats. Although the demonstration vehicles have been fairly simple, they have performed quite well. In comparison to analogous resonators that do not employ the deformation, the size can be reduced by a factor of four. Both the planar and wire-loaded cavity implementations of the Möbius resonator concept exhibited a dual-mode fundamental resonance at approximately half the frequency of a similar structure where the geometric deformation was not employed.

Future effort will be focused on quantifying the properties, examining the issues of tuning and coupling, and investigating the field patterns of the modes in various structures. More advanced resonators involving more complicated geometries are also being pursued. In particular, it is expected that the incorporation of dielectric loading could result in additional substantial savings in weight and volume.

Acknowledgement

The author wishes to acknowledge the numerous helpful discussions provided by C. Rauscher and D. Webb. This work was funded, in part, by the Office of Naval Research.

References

[1] Webster's N e wCollegiate Dictionary, G. & C. Merriam Company, Spingfield, MA, 1974.

[2] W. V. Tyminski, and A. E. Hylas, "A wide-band hybrid ring for UHF," *Proceedings of the I.R.E.*, vol. 41, pp. 81-87, Jan., 1953.

[3] D. Rubin, and D. Saul, "mm wave MICs use low value dielectric substrates," *Microwave Journal*, vol. 19, no. 11, pp. 35-39, Nov., 1976.

[4] B. R. Heimer, L. Fan, and K. Chang, "Uniplanar hybrid couplers using asymmetrical coplanar striplines," *IEEE Trans. on Microwave Theory and Tech.*, vol. 45, no. 12, pp. 2234-2240, Dec. 1997.

[5] W. W. Macalpine, and R. O. Schildknecht, "Coaxial resonators with helical inner conductor," *Proceeding of the I.R.E.*, vol. 47, pp. 2099-2105, Dec., 1959.

[6] S. J. Fiedziuszko, and R. S. Kwok, "Novel helical resonator filter structures," *1998 IEEE MTT-S Intern. Microwave Symp. Dig.*, vol. 3, pp. 1323-1326, June 7-12, 1998.