Bandpass Filters Using Dual-Mode and Quad-Mode Möbius Resonators

Jeffrey M. Pond, *Member, IEEE*, Shaojun Liu, and Nathan Newman

***Abstract*—Compact bandpass filters are being developed using Möbius wire-loaded cavity resonators. Initial results on tuned filters indicate that excellent filter characteristics can be attained in devices that are significantly smaller than traditional wire-loaded cavity technologies. A novel quad-mode Möbius resonator is presented which occupies the same volume as a dual-mode Möbius resonator. A four-pole bandpass filter is demonstrated using a single quad-mode Möbius resonator. Precisely controlled dielectric loading of dual-mode Möbius wire resonators has been implemented to realize bandpass filters. Two-pole and four-pole bandpass filters are demonstrated using one and two dual-mode dielectric-loaded Möbius resonators, respectively.**

***Index Terms*—Bandpass filters, cavity resonator filters, filters, microwave filters, resonators, topology.**

I. Introduction

BANDPASS filters can be realized from dual-mode Möbius resonators with intrinsic transmission zeros. The Möbius resonator utilizes a geometrical deformation of a transmission line to obtain a four-fold reduction in volume [1], [2]. Möbius resonators are the result of projecting a transmission line onto a Möbius strip, which is the prototypical nonorientable surface. Although traditionally referred to as one-sided surfaces, nonorientable surfaces are those for which the concept of left and right are globally nonsensical [3].

The nonorientable nature of the surface results in an apparent periodic alternation between left and right as the center circle of a Möbius strip is traversed. If a transmission line is projected onto a nonorientable surface with phasing required to sustain the electromagnetic oscillation along the path length associated with reversal of left and right, a resonant condition occurs with a volume reduction of a factor of four over conventional wire-loaded designs [1], [2].

An alternative way of visualizing the dominant modes is to consider that a transmission line projected onto a Möbius strip has a 180° twist of the geometry of the transmission line, which yields a phase reversal of the fields when the transmission-line ends are joined. The additional phase shift due to the twist yields a resonance when the transmission-line circumference is equal to a half wavelength. When realized with a twin conductor transmission line, the conductor can be visualized as the edge of the Möbius strip and an electric field flux line can be considered as the nonorientable surface.

Each of these Möbius resonator possesses two orthogonal modes and two intrinsic transmission zeros, as described previously [1], [2]. This paper extends the previous work by focusing on Möbius resonators inserted into cavities to realize bandpass filters. Such a resonator can be referred to as a Möbius wire-loaded cavity resonator even though the Möbius modes are determined primarily by the Möbius wire resonator rather than the cavity dimensions. With one of these resonators, it is possible to construct a compact two-pole two-zero bandpass filter with elliptic-type response. Using a novel quad-mode Möbius wire-loaded cavity resonator, a four-pole four-zero bandpass filter is implemented.

Additionally, more compact bandpass filters are realized using dielectric loading. Since the Möbius modes involve counter flowing currents in the wires, the electric field is concentrated largely between the wires with little field interaction with the cavity wall. Thus, by embedding the Möbius wire resonators in a dielectric, the circumference can, to first order, be reduced by the square root of the relative dielectric constant. Hence, the resonator volume is reduced by a factor equal to the relative dielectric constant. With this approach, dual-mode dielectric-loaded Möbius wire resonators are used singly and in pairs to construct two-pole and four-pole bandpass filters. Although implemented with a dielectric material with relative dielectric constant of 5.6, this approach can be extended to higher dielectric constant materials.

II. Dual-Mode and Quad-Mode Möbius Bandpass Filters

For ease of both fabrication and development of tuning, Möbius wire resonators were designed with a fundamental frequency of approximately 600 MHz. Möbius structures with a mean diameter of 8.128 cm were formed from 0.141-in-diameter copper wire. From a first principle calculation, where the mean circumference corresponds to a half wavelength, this yields a fundamental frequency of 587 MHz. The cylindrical aluminum cavity, into which the Möbius wire resonators were inserted, was designed with an internal diameter of 11.43 cm and a height of 3.175 cm. Glass-fiber reinforced dielectric boards, joined in the shape of an “x,” were used to position the Möbius wire resonator in the center of the cylindrical aluminum cavity. A dual-mode Möbius wire resonator and its dielectric support are shown in Fig. 1. Tuning screws in the top plate and sidewall of the cavity were used to adjust the resonant

Manuscript received March 31, 2001; revised August 23, 2001. This work was supported by the Office of Naval Research under Contracts N0001400WR220283 and N000140010550.

J. M. Pond is with the Naval Research Laboratory, Washington, DC 20375 USA (e-mail: j.m.pond@ieee.org).

S. Liu and N. Newman are with the Chemical and Materials Engineering Department, Arizona State University, Tempe, AZ 85287 USA.

Publisher Item Identifier S 0018-9480(01)10456-4.

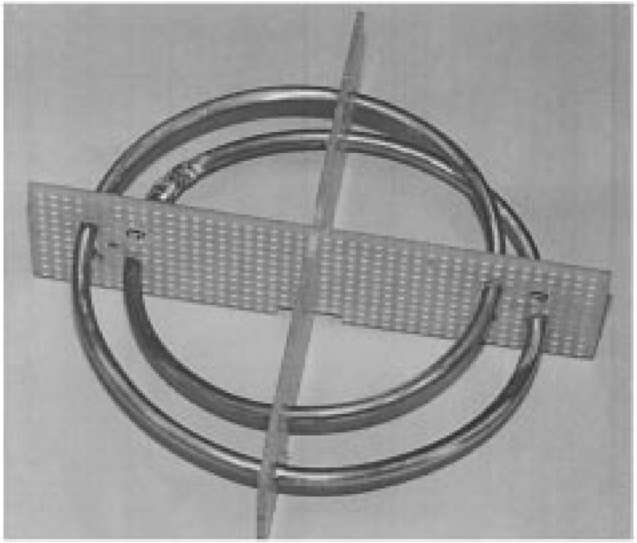


Fig. 1. Photograph of the dual-mode Möbius wire resonator that was placed in an 11.43-cm-diameter 3.175-cm-high cylindrical aluminum cavity.

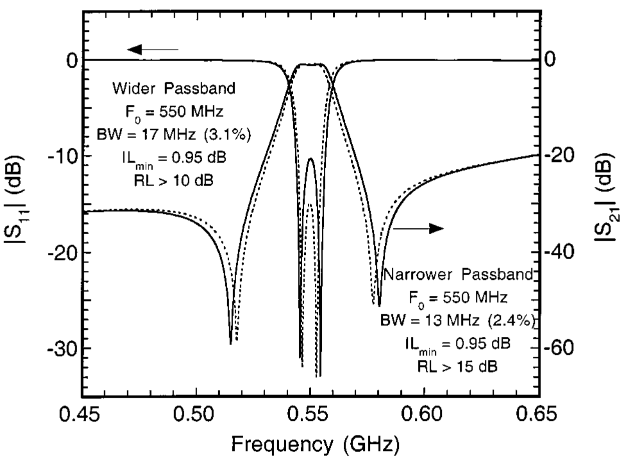


Fig. 2. Measured responses of the two-pole two-zero bandpass filter, consisting of a dual-mode Möbius resonator, when tuned for two different bandwidths.

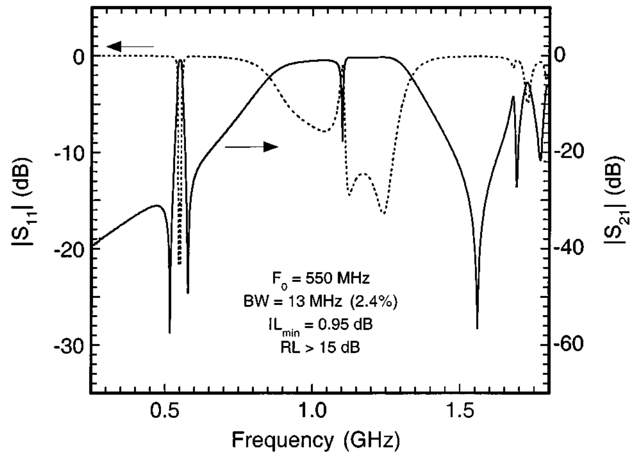


Fig. 3. Measured out-of-band response of the two-pole bandpass filter, consisting of a dual-mode Möbius resonator, when tuned for the narrower bandwidth.

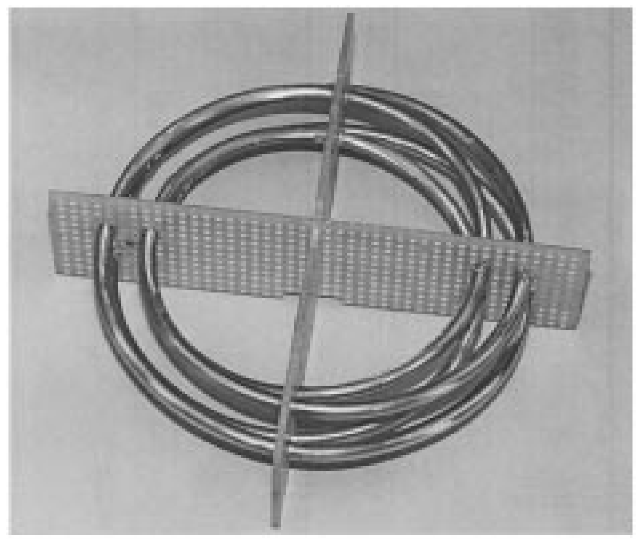


Fig. 4. Photograph of the quad-mode Möbius wire resonator that is placed in an 11.43-cm-diameter 3.175-cm-high cylindrical aluminum cavity.

frequencies and the coupling between modes in order to adjust the placement of the poles and zeros.

A high degree of symmetry in the passband shape in Fig. 2 around a 550-MHz center frequency is demonstrated. The solid and dashed lines are the result of two different tunings of the passband. In the first, a goal of 10-dB return loss was chosen. In the second, 15-dB return loss was selected. In both cases, the center frequency is maintained and a desirable near-symmetric elliptic-type filter response is obtained.

The limitation in the present implementation is revealed when examining the out-of-band response above the passband. As can be seen in Fig. 3, capacitive coupling to the Möbius wire resonator results in strong coupling to a mode at about twice the frequency of the fundamental Möbius mode. However, at this frequency, a Möbius mode does not exist. Instead, an anti-resonance occurs as a result of the 180° twist in combination with the wavelength being approximately equal to the circumference. The observed modes are a result of generalized even modes with respect to the cavity wall [4]. Techniques to minimize the excitation of these modes are being pursued.

A quad-mode resonator can be realized by interleaving two identical dual-mode Möbius wire resonators such that the two Möbius strips intersect each other along their center circles. In the terminology [3] of Lens Spaces, a dual-mode Möbius resonator is a substantial embedded surface in Lens Space $[2, 1]$ and two dual-mode Möbius resonators that intersect along their center circles are substantial embedded surface in Lens Space $[4, 2]$. By definition [3], Lens Space $[4, 2]$ degenerates to Lens Space $[2, 1]$.

Fig. 4 is a photograph of a quad-mode Möbius resonator consisting of two interleaved dual-mode Möbius resonators of the type shown in Fig. 1, where one resonator has been rotated about its center by 180° with respect to the other resonator. In a dual-mode Möbius resonator, the two dominant orthogonal modes each have one electric field maximum, which occur at "opposite" points on the circumference of the resonator from each other. By interleaving two resonators such that the electric

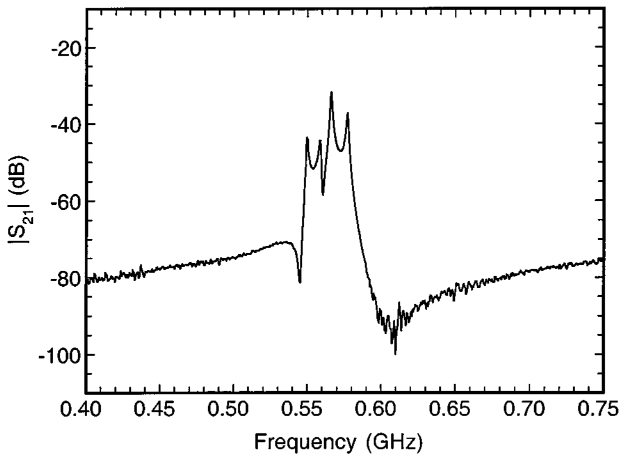


Fig. 5. Response of the quad-mode Möbius resonator when loosely coupled.

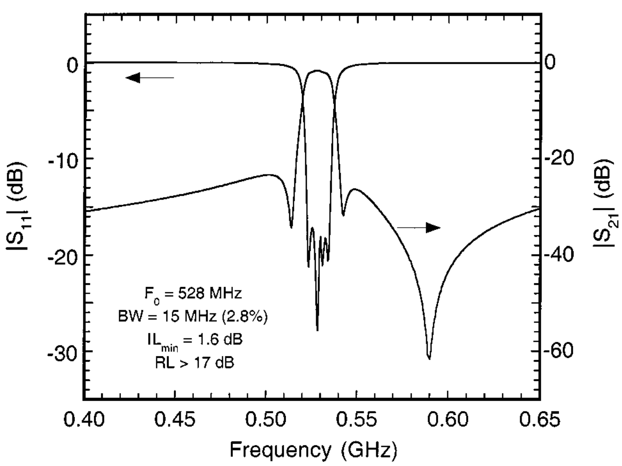


Fig. 6. Response of the four-pole bandpass filter, consisting of a single quad-mode Möbius resonator, tuned for best passband response.

fields between the wire pairs associated with each resonator are orthogonal, four modes will exist, two for each interleaved resonator.

When the structure in Fig. 4 is inserted into the same cavity and weakly coupled to the input and output ports, each of the four modes is clearly evident, as shown in Fig. 5. The substantial splitting between modes is due to the differences in parasitic couplings between the modes and the cavity walls, as well as inaccuracies in the machining and assembly of the structure. The intrinsic transmission zeros are also evident in Fig. 5.

Since the volume occupied by the quad-mode resonator is the same as that of the dual-mode resonator, an attempt was made to tune a quad-mode resonator with a set of tuning screws designed to tune a dual-mode structure. Not surprisingly, this attempt was less than totally successful [5]. However, by adding some tuning screws to the bottom cover of the cylindrical cavity, a much better bandpass response could be obtained, as shown in Fig. 6. Unlike previously reported results [5], no lumped-element capacitances were used to modify the mode coupling. All tuning was accomplished with the metallic tuning screws. Input and output port couplings were controlled by nylon screws, which adjusted the gap between the input and output antennas and the Möbius wire. The resultant filter performance is substantially improved. The passband is 15-MHz wide and is centered at 528 MHz, which corresponds to a 2.8% passband. The return loss is greater than 17 dB and the minimum insertion loss is 1.6 dB. The location of the four poles can be readily seen in $|S_{11}|$.

The location of the transmission zeros could not be independently controlled to the degree desired. Specifically, although there should be four transmission zeros, only three are readily apparent in Fig. 6. Two of the transmission zeros are colocated at 515 MHz, just below the passband. Attempts to separate these zeros resulted in degradation of the passband shape, indicating that further refinement in the control and placement of the tuning screws is required.

III. Dielectric-Loaded Möbius Bandpass Filters

A number of dielectric Möbius resonator test structures were initially fabricated [5] using alumina ceramics, in conjunction with small diameter gold or platinum metal wire. A parallel loop geometry was employed since it can be accurately fabricated and modeled. Alumina was chosen for initial testing because of its excellent microwave performance, high thermal conductivity, and relatively large dielectric constant.

Although initial measurements using these resonators without tuning screws showed promise, for ease of fabrication during development, Macor[1] was used for the dielectric. Macor is an easily machinable glass–ceramic dielectric with a relative dielectric constant of 5.6 at 8.5 GHz. The dielectric loss tangent has been reported as 7.1×10^{-3} at 8.5 GHz, which, although not nearly as low loss as alumina, is considered acceptable for the present development work on these resonators.

Dielectric-loaded Möbius wire resonators were fabricated using Macor, in conjunction with 0.25-mm-diameter gold wire. The structures consisted of two parallel Au loops recessed into 0.38-mm-deep circular grooves in the Macor ceramic. The loops are 0.74 cm in diameter and are separated by a distance of 0.31 cm. The Au wire was configured into the Möbius geometry by connecting the loops through 0.4-mm-diameter via holes. The cylindrical resonators have an overall diameter of 1.27 cm and a thickness of 0.38 cm. A 0.22-cm-diameter hole was drilled through the cylindrical axis of the Macor resonators through which a metallic rod could be press fit.

Fig. 7 shows a cross-sectional representation of a Macor dual-mode Möbius resonator in a cylindrical copper cavity. The dimensions shown in Fig. 7 are not to scale but help to illustrate the approach implemented. The metallic rod press-fit through the Macor resonator was placed axially in the cavity and extended through the cavity end walls. The resultant axial rotation facilitates the adjustment of the coupling between the two orthogonal modes as well as the coupling to the input and output ports. The input and output leads are 0.085-in-diameter coaxial cable whose coupling to the resonators could be adjusted by sliding the cable into the cavity. Thin dielectric spacers were used to preclude electrical contact between the input and output antennas and the Möbius wire recessed

[1]Macor is a product of Corning, Inc., Corning, NY, and is available from distributors. Distributor websites ([Online]. Available: http://www.accuratus.com/MACOR.htm and http://www.technicalglass.co.uk/macordata.html) contain physical, mechanical, and electrical specifications.

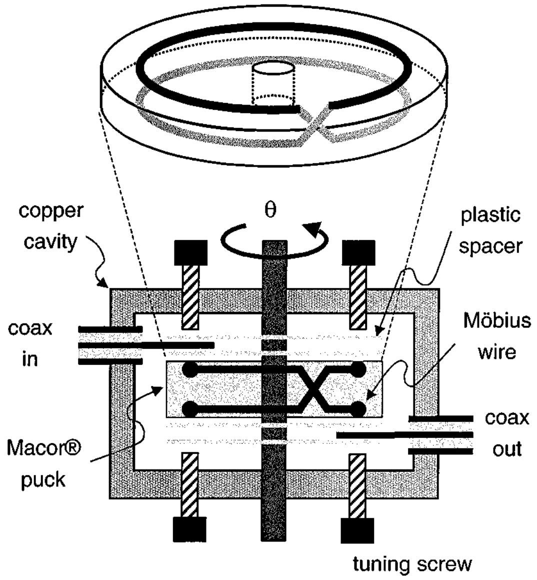


Fig. 7. (*top*) Perspective drawing of the dielectric-loaded Möbius wire resonator showing the Macor puck and the embedded gold wire with the cross over accomplished with two vias. (*bottom*) Edge view of a two-pole filter consisting of a dual-mode dielectric-loaded Möbius wire resonator in a copper cavity.

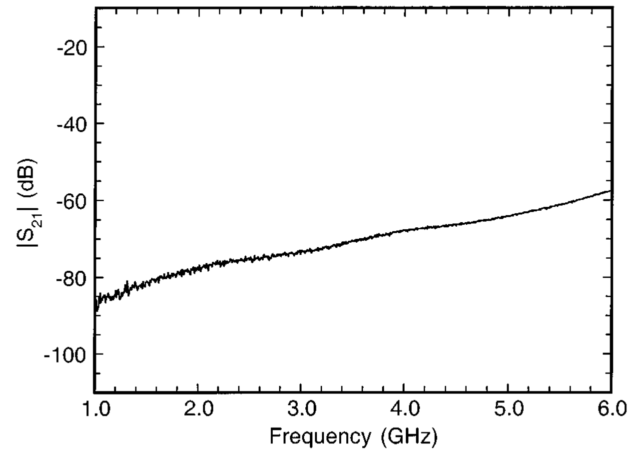


Fig. 8. The 1.0–6.0-GHz response of a "control" Macor cylinder without an embedded Möbius wire resonator.

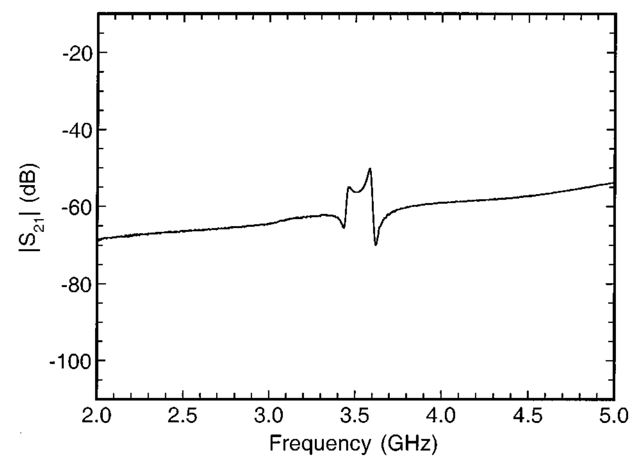


Fig. 9. Weakly coupled response of the dielectric-loaded Möbius wire resonator shown in Fig. 7.

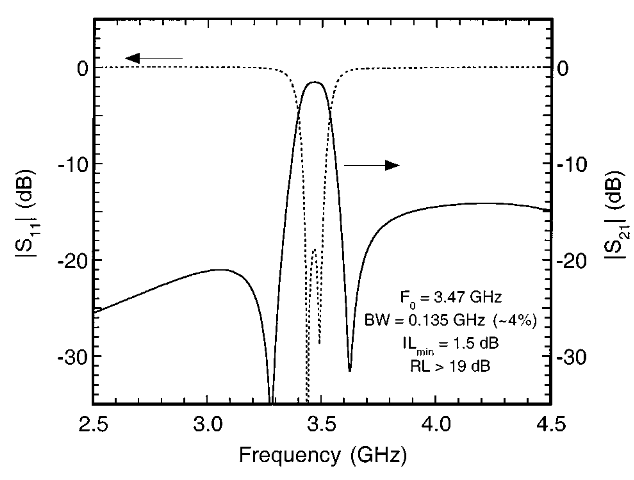


Fig. 10. Tuned two-pole two-zero bandpass filter response of the dielectric-loaded Möbius wire resonator filter shown in Fig. 7.

into the Macor cylinder and the end-wall tuning screws. The end-wall tuning screws were used to tune the input and output port coupling as well as the location of the transmission zeros. Unlike the filters demonstrated in the previous section, no sidewall tuning screws were available with this cavity. The gray lines represent the path of the embedded Möbius wire including the "twist" which is accomplished by the two via holes mentioned previously.

For convenience, the copper cavity in which the resonator was suspended was larger than required to accommodate the Macor-loaded Möbius resonator. Since the relative dielectric constant of Macor is 5.67, the Möbius loop diameter, assuming 100% of the fields are confined to the Macor, would yield an expected resonance at 2.71 GHz. To verify that there are no other resonances in the range of interest, an identical Macor cylinder, minus the embedded Möbius wire resonator, was mounted in the cavity in an identical fashion. Fig. 8 shows that there are no modes excited in the 1.0–6-0 GHz frequency range.

With the Macor-loaded Möbius wire resonator inserted into the copper cavity, as shown in Fig. 7, the weakly coupled response was measured and is shown in Fig. 9. The dual modes and two transmission zeros are evident and are centered at 3.5 GHz. This is higher than the 2.71-GHz resonance that could be expected if the fields were entirely contained in the Macor dielectric. However, given the air gaps that could be expected, since the gold wire has been laid in a machined grove, and since the wires are flush with the surface of the Macor, the resultant effective relative dielectric constant of 3.4 is quite reasonable.

When the input and output port couplings are adjusted properly along with the end-plate tuning screws and the rotational orientation, θ, of the Macor-loaded Möbius wire resonator, a reasonably symmetric elliptic-type bandpass filter response can be realized as shown in Fig. 10. The center frequency is shifted slightly lower to 3.47 GHz as the input and output port couplings have been increased. The passband is 135 MHz corresponding to approximately a 4% bandpass. The return loss is greater than 19 dB and the minimum insertion loss in the passband is 1.5 dB. Improvements in out-of-band isolation are still needed and should be aided by improved input and output port

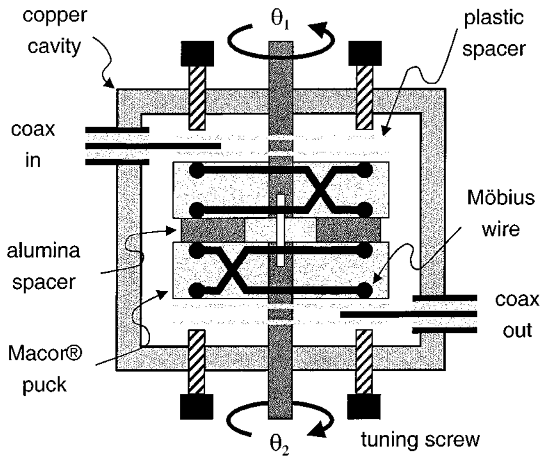


Fig. 11. Edge view of a four-pole filter consisting of two cascaded dual-mode dielectric-loaded Möbius wire resonators in a copper cavity.

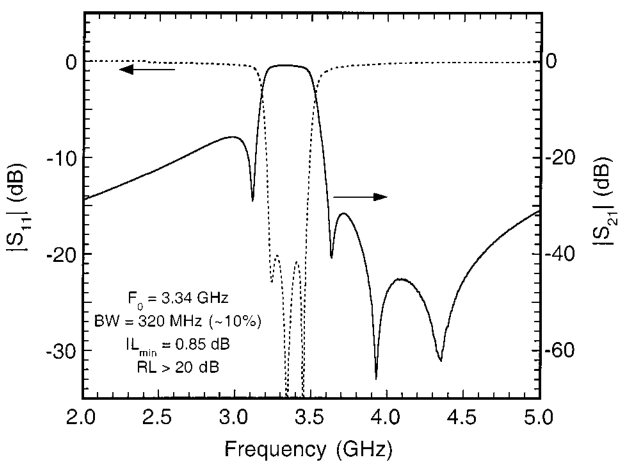


Fig. 12. Tuned four-pole four-zero bandpass filter response of the dielectric-loaded Möbius wire resonator filter shown in Fig. 11.

coupling which minimizes direct input port to output port coupling.

A four-pole bandpass filter was realized using two cascaded dual-mode dielectric-loaded Möbius resonators. The structure fabricated is shown in Fig. 11. It is a straightforward extension of the design shown in Fig. 7. The dielectric-loaded Möbius wire resonators were nominally identical to the one described previously. The separation between resonators was held fixed by the alumina spacer shown in the cross-sectional diagram. Coupling between the two resonators was limited to the relative θ orientations of the two resonators, which could be rotated on their axes independently. Otherwise, tuning was limited to the endplate tuning screws and the insertion depth of the input and output port coupling antennas into the copper cavity.

Within the tuning limitations discussed, the four-pole filter was tuned for the best obtainable passband shape. The results of these efforts are shown in Fig. 12. The 320-MHz ($\sim$10%) passband is centered at 3.4 GHz. The minimum passband insertion loss is 0.85 dB while the return loss is greater than 20 dB. Although the poles are not ideally positioned within the passband, the overall passband shape is quite respectable. Within the limitations imposed by the tuning available, it was impossible to position two transmission zeros both above and below the passband to present a more symmetric response.

IV. Conclusion

Two-pole and four-pole bandpass filters with elliptic-type responses have been demonstrated using Möbius wire resonators. A symmetric 2.5% bandpass filter centered at 550 MHz with two poles and two transmission zeros has been developed using a tuned dual-mode Möbius resonator configuration. The minimum insertion loss was less than 1 dB and the return loss was greater than 15 dB. A novel quad-mode resonator, consisting of two interleaved dual-mode resonators, was introduced as a means of realizing a more compact four-pole filter. An asymmetric 2.8% four-pole bandpass filter centered at 528 MHz was tuned using this quad-mode resonator geometry. The minimum insertion loss was 1.6 dB and the return loss was greater than 17 dB. Although the filter characteristics achieved represent a substantial improvement over those achieved previously [5], further improvement in filter tuning will be required to obtain an ideal response.

The incorporation of dielectric loading of Möbius wire resonators was successfully demonstrated. With this approach, a very compact resonator volume can be achieved since the size reduction due to dielectric loading and the Möbius resonator concept are synergistic. Dielectric-loaded dual-mode Möbius resonators are used singly and in cascade to realize two-pole and four-pole bandpass filters, respectively, with elliptic-type responses. Excellent passband responses were obtained given the limited tuning available. A 4% two-pole two-zero bandpass filter, centered at 3.47 GHz, was constructed with a single resonator and exhibited 1.5-dB minimum insertion loss and greater than 19-dB return loss with a nearly symmetric passband. In addition, a 10% four-pole four-zero bandpass filter, centered at 3.34 GHz, was fabricated from two cascaded dual-mode resonators. This filter possessed a minimum insertion loss of less than 1 dB and a return loss of greater than 20 dB.

These encouraging results indicate that this technology can be used to develop compact high-performance filters. Future efforts will focus on developing methods to suppress the excitation of the higher order modes and improve tuning. Additionally, losses can be reduced in the dielectric-loaded structures by using larger diameter embedded Möbius wire resonators and the use of lower loss dielectrics. Further significant reductions in size are expected to be realized by incorporating state-of-the-art microwave dielectric ceramics.

Acknowledgment

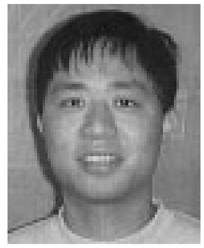
The authors would like to thank Dr. C. Rauscher, Naval Research Laboratory, Washington, DC, and Dr. D. C. Webb, Naval Research Laboratory, Washington, DC, for their encouragement and useful discussions on microwave filters. Prof. J. S. Carter, Department of Mathematics, University of South Alabama, Mobile, provided clarification on some aspects of topology. The tutelage of R. Hershtig, K&L Microwave Inc., concerning the practical aspects of tuning filters is greatly appreciated.

REFERENCES

[1] J. M. Pond, “Möbius filters and resonators,” in *IEEE MTT-S Int. Microwave Symp. Dig.*, vol. 3, June 2000, pp. 1653–1656.
[2] ——, “Möbius dual-mode resonators and bandpass filters,” *IEEE Trans. Microwave Theory Tech.*, vol. 48, pp. 2465–2471, Dec. 2000.
[3] J. S. Carter, *How Surfaces Intersect in Space: An Introduction to Topology*, 2nd ed. Singapore: World Scientific, 1995.
[4] J. M. Pond, “Modes of a Möbius resonator,” *Electron. Lett.*, vol. 37, no. 13, pp. 836–838, June 2001.
[5] J. M. Pond, S. Liu, and N. Newman, “Dual-mode and quad-mode Möbius bandpass filters,” in *IEEE MTT-S Int. Microwave Symp. Dig.*, vol. 3, May 2000, pp. 1771–1774.

Jeffrey M. Pond (S’82–M’82) received the B.S. degree in electrical engineering from Michigan State University, East Lansing, in 1978, and the M.S. and Ph.D. degrees in electrical engineering from The University of Michigan at Ann Arbor, in 1979 and 1982, respectively.

While at The University of Michigan at Ann Arbor, he was a Graduate Research Assistant in the Radiation Laboratory. In September 1982, he joined the Electronics Science and Technology Division, Naval Research Laboratory, Washington, DC, where he is a member of the Microwave Technology Branch’s Solid-State Circuits Section. In addition to his interest in microwave filters, his research has included cryogenic microwave measurements and electronics and microwave measurements and applications of superconductors and ferroelectrics.



Shaojun Liu was born in Hunan, China. He received the B.S degree in materials science and engineering and the M.S degree in powder metallurgy from the Central South University of Technology, Changsha, China, in 1993 and 1996, respectively, and is currently working toward the Ph.D. degree in materials science at Arizona State University, Tempe.

His research interests are the synthesis, growth, and characterization of high-performance microwave dielectrics.

Nathan Newman was born in Berkeley, CA. He received the B.S. degree in biomedical/electrical engineering from the University of Southern California, Los Angeles, in 1981, and the M.S. and Ph.D. degrees from Stanford University, Stanford, CA, in 1983 and 1987, respectively.

He is currently a Professor in the Chemical and Materials Engineering Department, Arizona State University, Tempe. He was an Associate Professor at Northwestern University, Evanston, IL, and a Member of the Technical Staff at the University of California at Berkeley, Lawrence Berkeley Laboratory, Stanford University, and Conductus Inc. His research has focused on novel solid-state materials for microwave, photonic, and high-speed applications. His current work involves synthesis, characterization, and modeling of III–N semiconductors, high-Q microwave dielectrics, superconductor SNS junctions for 100+-GHz digital logic heterojunction, metal/III–V semiconductor interfaces, and nonstoichiometric transition–metal nitrides near the metal–insulator transition.