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RESONATORS ON NON-ORIENTABLE SURFACES

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Non-orientable surfaces are a class of surfaces studied in topology where the notions of left and right dont make sense globally. The Moebius strip is the most well known non-orientable surface and is indeed the prototypical non-orientable surface since every non-orientable surface has a Moebius strip as a subsurface. While repeatedly traversing the center circle of a non-orientable surface, left and right appear to alternate with each traverse of the center circle. Similarly, a traveling wave on a transmission line experiences periodic reversals in polarity. By projecting a transmission line onto a non-orientable surface and phasing the electromagnetic oscillation with the path length associated with reversal of left and right a resonant condition occurs. The resonance occurs when the path length is a half wavelength even though the transmission line is smoothly closed on itself.

This concept can be most easily visualized and realized experimentally when the non-orientable surface is a Moebius strip. A two-wire transmission line can be used to construct such a resonator where the edge of the Moebius strip is the conductor and an electric field contour defines the surface. By bending a finite length two-wire transmission line back on itself while twisting the transmission line 180 degrees about its axis and joining the conductors together, the fundamental resonance will exist at half the frequency of a conventionally connected ring-resonator. Higher order resonances will occur when the path length is an odd integer multiple of a half wavelength. These resonators have been realized in both planar and 3D form where their dual-mode characteristics have been experimentally verified (J.M. Pond, 2000 IEEE MTT-S, Int. Microwave Symp. Dig., vol. 3, pp. 1653-1656, June 2000).

The mode structures of transmission lines on projected planes associated with non-orientable substantial surfaces in Lens Spaces (J.S. Carter, How Surfaces Intersect in Space: An Introduction to Topology, World Scientific, Singapore, 1995) will be discussed. Examples of such surfaces are two Moebius strips which intersect each other along their center circles and a single Moebius strip which intersects itself along its center circle.